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Design of State Feedback Gain Matrix for DC Motor Control Based on Damping Ratio and Natural Frequency

Muawia Mohamed Ahmed Mahmoud
AL Neelain University, Faculty of Engineering, Control Engineering Department,
Khartoum, Sudan
Muawia15858@hotmail.com

Abstract: ***This paper aims to describe a method of designing a state vector to place the poles of the characteristic equation of the closed loop of the speed transfer function of the DC motor. The method uses the open loop of the speed transfer function or its controllable canonical form to compare it with the desired closed loop transfer function. The desired characteristic equation of the closed loop is specified by a desired damping ratio and natural frequency.***

Keywords: Transfer function, State-space, State vector, Damping ratio, Natural frequency.

1. Introduction

Dynamic systems are mathematically modeled in state-space form. Starting from the differential equation of the system, the transfer function is derived. DC motors are commonly used to provide rotary (or linear) motion to a variety of electromechanical devices and servo systems. In most applications the speed or position of the shaft of these motors must be accurately controlled. In order to design such velocity and position control systems it is necessary to obtain, analytically or experimentally, a mathematical model for the motor or system to be controlled. If the system is predominantly linear, a suitable model is given by its Transfer Function. For the armature controlled DC motor position control, the transfer function of the system is derived between the angular position at the output and the applied voltage to the armature of the DC motor as the input. This approach normally produces a third order system when none of its parameters was ignored. Depending on the system's parameters, the characteristic equation of the system determines the system behavior. The damping ratio (ζ) and the natural frequency (ω_n) are important parameters in the characteristic equation of the transfer function since the overshoot, settling time and other transient response are strongly related to these two parameters. By specifying some desired values of ζ and ω_n , the desired output response could be obtained [1,2].

An armature current DC motor with the following parameter is considered

J=0.01; b=0.1; K=0.01; R=1; L=0.5;

Where J is equivalent moment of inertia reflected at the motor shaft.

b = equivalent viscous coefficient reflected at the motor shaft.

K = motor torque constant

R= windings resistance, and

L= inductance

The state-space system of the system is:

A= [-b/J K/J
 -K/L -R/L];

B= [0
 1/L];

C= [1 0];

D=0;

Equation (1) shows the angular position transfer function of DC motor after substitution of its parameters[3] .

The transfer function of a typical permanent magnet DC motor has the general form:

$$G(s) = \frac{2}{S(S^2+12S+20)} \quad (1)$$

To design the gain matrix, the phase canonical form should be found.

The matrices of the phase canonical state-space of the DC motor system are defined as:

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -20 & -12\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad C=[\,2\ \ 0\ \ 0\,],\quad D=0 \quad (2)$$

2. Gain Matrix Design

The state equation of the system described by the state-space representation in the form shown in equation (3) can be described by the block diagram shown in figure (1) [4].

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \qquad (3)$$

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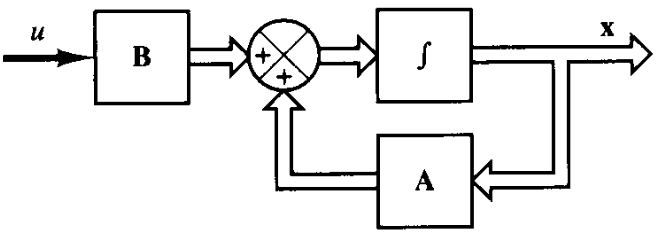

Figure (1): Block diagram of the open loop state equation

The characteristic equation of the transfer function of the DC motor is :

$$S^3 + 12S^2 + 20S + 0 = 0 \qquad (4)$$

The system has three poles with the values:
$P_1=0$, $P_2=-10$, $P_3=-2$.

Now it is required to place these poles in other locations so as to give a damping ratio (ζ) of 0.8 , and natural frequency ω_n =6. A gain matrix K must be inserted in a feedback loop to force the system to have the criteria specified. Figure (2) illustrates the block diagram of the system with the gain matrix feedback loop.

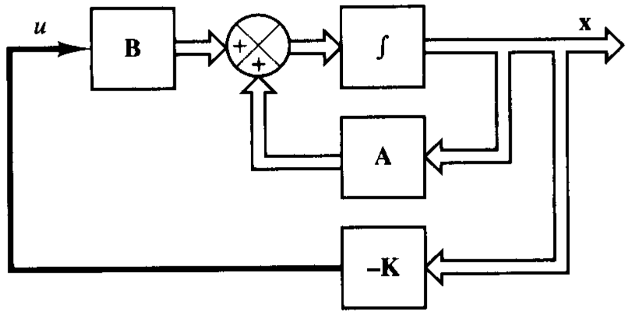

Figure (2): Closed loop with the gain matrix

The input (u) of the state equation is changed to:

$$u = -\mathbf{K}\mathbf{x} \qquad (5)$$

And the state equation will be:

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{BK})\mathbf{x}(t) \qquad (6)$$

The stability and transient-response characteristics are then determined by the eigen values of matrix (A-BK)[5,6].

Assume it is required to make the closed characteristic equation in the form shown in equation (4) with the above given values of damping ratio and natural frequency.

$$(S^2 + 2\zeta\omega_n + \omega^2_n)(S+\zeta\,\omega_n) \qquad (7)$$

with $\zeta = 0.8$ and $\omega_n = 6$

. Substituting the given values of the damping ratio and the natural frequency in the characteristic equation yields

$$S^3 + 14.4\,S^2 + 82.1S + 172.8 = 0 \qquad (8)$$

Recalling equation (3)

$$BK = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_1 & k_2 & k_3 \end{matrix} \qquad (9)$$

and

$$A - BK = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k1 & (-20-k2) & (-12-k3) \end{matrix} \qquad (10)$$

Now the characteristic equation from equation (7) is calculated as:

$$S^3 + (12+k_3) + (20+k_2) + k_1 \qquad (11)$$

Comparing the new characteristics with k matrix yields:

12+k3 = 14.4 from which $k_3 = 2.4$
20+k2= 82.1 from which $k_2 = 60.1$
$K_1 = 172.8$

So the gain matrix of K = [172.8 60.1 2.4] should be inserted in the feedback loop to obtain the desired behavior of the system.
Table (1) shows the resultant ζ and ω_n .

3. Results Discussion

To ensure that the gain matrix has given the desired response, Matlab step command could be used for the feedback system.
The overall transfer function of the system is:

$$G_f(s) = \frac{2}{S3 + 14.4S^2 + 82.1S + 172.8} \qquad (12)$$

To find the poles of the system, Matlab command pole could be used which gives:
$P_1 = -4.8037 + j3.6028$
$P_2 = -4.8037 - j3.6028$
$P_3 = -4.7926$

The zero-pole form of the G_f could be written as:

$$G_{zp}(s) = \frac{2}{(S+4.8+j3.6)(S+4-j3.6)(s+4.79)} \qquad (13)$$

To check that the specified damping ratio and natural frequency were obtained , Matlab command (damp) is used.
The result of the command yields the values described in table (1).
From the table, the damping ratio of 0.8 was obtained for the two conjugate poles together with the corresponding natural frequency of 6 as shown in the table.

Table (1): Achieving the specified ζ and ω_n of the system.

Eigenvalue	Damping	Freq. (rad/s)
-4.79	1.00e+000	4.79e+000
-4.80+ 3.6j	8.00e-001	6.00e+000
-4.80+ 3.6j	8.00e-001	6.00e+000

References

4. Conclusion

In my paper I have designed a state feedback gain matrix for the DC motor with specified criteria. The desired behavior of the system was that to have 0.8 damping ratio and 6 rad/s of natural frequency. Rather than pole placement method, the method here concentrates on the required system behavior. In pole placement the designer determines the required pole locations first, then proceed with the steps needed. The main benefit of designing the state feedback matrix based on the transient response is that system starting problems could be expected and well managed. As the results shown, the required damping ratio and natural frequency were obtained after adding the state feedback gain matrix. The phase canonical state-space representation must be found first from the system's differential equation or the transfer function. From this representation the new characteristic equation of the designed system was achieved, and the row vector of the gain matrix elements was found.

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